Homework 4

1. Find the $\frac{dy}{dx}$ by implicit differentiation

$$x^4y - 8xy + 3xy^2 = 9$$

2. Find the value of the derivative (if it exists) at the indicated extremum. (If an answer does not exist, enter DNE.)

$$f(x) = -7x\sqrt{x+1}$$

3. $f(x) = \begin{cases} 4 - x^2, x \leq 0 \\ -2x, x > 0 \end{cases}$

(a) Find the critical numbers of f

(b) Find the open intervals on which the function is increasing or decreasing

(c) Apply the First Derivative Test to identify all relative extrema

Sol：

1.

$$x^4y - 8xy + 3xy^2 = 9$$
$$x^4y' + 4x^3y - 8xy' - 8y + 6xyy' + 3y^2 = 0$$
$$(x^4 - 8x + 6xy)y' = 8y - 4x^3y - 3y^2$$
$$y' = \frac{8y - 4x^3y - 3y^2}{(x^4 - 8x + 6xy)}$$

2.

$$f(x) = -7x\sqrt{x+1} = (-7x)(x+1)^{\frac{1}{2}}$$
$$f'^{(x)} = (-7x)\left[\frac{1}{2}(x+1)^{-\frac{1}{2}}\right] + (-7)(x+1)^{\frac{1}{2}}$$
$$= \frac{-7}{2}(x+1)^{-\frac{1}{2}}\ [x + 2(x+1)]$$
$$= \frac{-7}{2}(x+1)^{-\frac{1}{2}}\ (3x+2)$$
$$x = -\frac{2}{3}\ \rightarrow f'\left(-\frac{2}{3}\right) = 0$$

3.

(a)

$$f(x) = \begin{cases} 4 - x^2, x \le 0 \\ -2x, x > 0 \end{cases}$$
$$f'(x) = \begin{cases} -2x, x < 0 \\ -2, x > 0 \end{cases}$$

Critical number：$x = 0$

(b)

Test intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of $f'(x)$	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $(-\infty, 0)$

Decreasing on: $(0, \infty)$

(c)

要使用 First Derivative Test 時，$f(x)$要是連續的(題目為不連續)，所以該題送分。